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Mathematically unsound exercises on “logical series”

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Abstract— Even if they have been criticized for decades, exercises based on « logical series », or “number patterns”, where students in primary, and even in secondary schools, and adults in recruitment process, are asked to guess what are the numbers after a given series of whole numbers, are still used with mathematically incorrect instructions and rules to assess them. A multitude of websites offer such exercises with explanations on response strategies, mainly based on a misunderstanding of mathematical series.

In this article it will be demonstrated why the bases of this kind of exercises are, generally, not mathematically correctly based. Therefore, this type of exercise should not be used to assess a mathematical skill – or logic – whether in the school setting or in professional recruitment. Mathematically the answers expected by the authors of these exercises are correct, so it is not embarrassing to consider them as such. On the other hand, refusing any other answer as erroneous is mathematically unfounded and unfair to the person assessed.

Some clues will be given on how to write the instructions to avoid such trap and, for instance, to fit in the international objectives as those given in the Global Proficiency Framework for Mathematics, for primary schools.

Index Terms—*Logical series, Math assessment, Pattern numbers.*

I. INTRODUCTION

The adjective «logic» is commonly used to characterize games where the player is supposed to reason according to certain logic to win.

It turns out that sometimes the term “logic” is a misnomer for mathematical reasoning. This is the case of exercises that require the player to complete a sequence of numbers. In fact it is a matter of guessing which transformation the author used to pass from one number to another, checking that it works on all the given numbers and then finally applying this same transformation to the last given term to deduce the following numbers.

However, without specific information about it, there is no uniqueness of this transformation. It is therefore a riddle and not a mathematical reasoning. It does not matter if it is a game without consequences, but it becomes so if this exercise is supposed to assess “logical” skills in the school setting or in professional recruitment.

Anglophones use the term “number pattern” to characterize some of these exercises. This emphasizes transformation as a model that applies on a recurring basis. This is in fact what we find in the objectives of the Global Proficiency Framework for Mathematics.

II. ANALYSIS OF AN EXAMPLE

To illustrate our point, let us take the example of this exercise of a 9th-grade French mathematics textbook in a chapter devoted to literal calculus, entitled “For the experts”:


11 Pour les experts
Voici le début d’une suite : 1 ; 2 ; 5 ; 10 ; 17 ; 26 ; 37 ; 50… Écris les trois nombres suivants et trouve l’expression littérale qui permet de construire cette suite.

For the experts

Here is the beginning of a series:
1 ; 2 ; 5 ; 10 ; 17 ; 26 ; 37 ; 50…
Write the 3 next numbers and find the literal expression that allows to build this series

Let us present this exercise in a mathematical form:

Be U a series such as:

U1 = 1, U2 = 2, U3 = 5, U4 = 10, U5 = 17, U6 = 26, U7 = 37 and U8 = 50

Find U9, U10 and U11 and give the literal expression of Un.

We note the expression “give the literal expression” and not "give a literal expression", which implies that for the author of the exercise there is such an expression and that it is unique.

However, from a mathematical point of view, knowing the values of U on {1, 2, 3, 4, 5, 6, 7, 8} does not provide any information on the values that U can take elsewhere.

For example, one could give the following answer:

$U1 = 1, U2 = 2, U3 = 5, U4 = 10, U5 = 17, U6 = 26, U7 = 37$ and $U8 = 50$

and for any integer n greater than or equal to 9, $Un = 0$. The answer would therefore be:

$U9 = U10 = U11 = 0$

Another example is:

$U1 = 1, U2 = 2, U3 = 5, U4 = 10, U5 = 17, U6 = 26, U7 = 37$ and $U8 = 50$

and for any integer n greater than or equal to 9, $A = n^2$. The answer would therefore be:

$U9 = 81, U10= 100, U11= 121$

Concretely, if in the statement of the problem no information is given to limit the field of possibilities there are infinity of possible answers.

In fact, this type of exercise is designed with a «playful» approach: the author chooses a more or less complex function, which he/she applies to the first integers and which the «player» (student or adult) must guess what the author has imagined. The only way to win this game is to guess the author's idea. All answers are mathematically correct but only those expected by the author are accepted.

Therefore, this type of exercise should not be used to assess a mathematical skill – or logic – whether in the school setting or in professional recruitment.

In the previous example, what can one imagine in the author's head that "looks" like a repeated mathematical operation?

Let us start from what is given in the statement:

$U1 = 1, U2 = 2, U3 = 5, U4 = 10, U5 = 17, U6 = 26, U7 = 37$ and $U8 = 50$

These values can be rewritten differently:

$U1 = 1 = (1 – 1)^2 + 1$; $U2 = 2 = (2 – 1)^2+1$; $U3 = 5 = (3 – 1)^2 + 1$; $U4 = 10 = (4 – 1)^2 + 1$;

$U5 = 17 = (5 – 1)^2 + 1$, $U6 = 26 = (6 – 1)^2 + 1$; $U7 = 37 = (7 – 1)^2 +1$ et $U8 = 50 = (8 – 1)^2 + 1$

For n between 1 and 8 we have the relation: $Un = (n-1)^2 + 1$.

If this is what the author of the exercise – or the person evaluating the exercise – has imagined, then the expected "right" answers are:

$U9 = (9 – 1)^2 + 1 = 65$

$U10 = (10 – 1)^2 + 1 = 82$

$U11 = (11 – 1)^2 + 1 = 101$.

Mathematically these answers are correct, so it is not embarrassing to consider them as such. On the other hand, refusing any other answer as erroneous is mathematically unfounded and unfair to the person assessed.

Many sites, including English-speaking sites, offer activities to learn how to solve pattern numbers exercises, as well as on YouTube. It should be noted that they do mention the existence of several models available but that they do not call into question the uniqueness presupposed of the model for a given exercise.

For instance:

"Here are some of the key points to be remembered when dealing with patterns.

- Number patterns are not restricted to a few types. They could be ascending, descending, multiples of a certain number, or series of even numbers, odd numbers, etc.

- Learning patterns enhances our capability to observe patterns. Observing a pattern pushes us to think and identify the rule which can continue the pattern." [2]

III. A MATHEMATICALLY CORRECT CONSTRUCTION METHOD

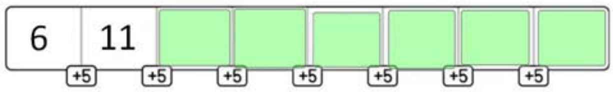
If we refer to the Global Proficiency Framework for Mathematics [1] we find this descriptor for Grade 4:

Describe numerical patterns that increase or decrease by a constant value with a simple rule, and use this information to identify a missing element or extend the pattern (e.g., describe the pattern 6, 9, 12, 15 as going up by threes; identify the missing element in the pattern 3, 7, 11, , 19; extend the pattern 6, 11, 16, 21).

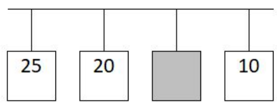
It is therefore a question of having students work on a type of explicit model, for example for grade 4, of going from one number to the next by adding or subtracting the same number.

We can therefore have exercises where this number is given – example 1 below – or to be calculated, as in example 2, but in both cases it is a mathematical reasoning and not a riddle.

Example 1: Fill in the green boxes using the same pattern



Example 2: Look at these numbers, they are 25, 20, and 10. What number goes in this empty box if you move from one number to the next, subtracting the same number each time?



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[2] Cuemath. https://www.cuemath.com/geometry/patterns/.